ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

① On écrit $\tilde{Z} = \frac{Z}{Z_0} = \frac{1}{Z_0}\frac{V(z)}{I(z)} \overset{(*)}{=} \boxed{\frac{1+\rho e^{j\psi} e^{j2\beta z}}{1-\rho e^{j\psi} e^{j2\beta z}} = 1 + j\tilde{X}_a}$

$\Rightarrow 1 + j\tilde{X}_a = \frac{1+\rho e^{j\theta}}{1-\rho e^{j\theta}} \qquad \theta = \psi + 2\beta z$

$\Rightarrow 1 + j\tilde{X}_a = \frac{1-\rho^2}{1-2\rho\cos\theta+\rho^2} + j\frac{2\rho\sin\theta}{1-2\rho\cos\theta+\rho^2}$

$\Rightarrow \frac{1-\rho^2}{1-2\rho\cos\theta+\rho^2} = 1 \Rightarrow 1-\rho^2 = 1-2\rho\cos\theta+\rho^2 \Rightarrow \theta = \cos^{-1}(\rho)$

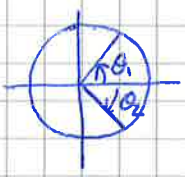
$\Rightarrow \psi + 2\beta z_1 = \cos^{-1}(\rho) \Rightarrow \boxed{z_1 = \frac{\cos^{-1}(\rho) - \psi}{2\beta} = \frac{\lambda}{4\pi}\left[\cos^{-1}(\rho) - \psi\right]}$

* Rq: 2 sol for $\cos^{-1}(\rho) = \theta$: $0 \le \theta_1 \le \pi/2$ et $-\pi/2 \le \theta_2 \le 0$

$\Rightarrow$ calculer les deux et se souvenir que $z_1 < 0$ (retirer $\frac{\lambda}{2}$)

$\Rightarrow \tilde{X}_a = \left.\frac{2\rho\sin\theta}{1-2\rho\cos\theta+\rho^2}\right|_{\rho=\cos\theta} = \frac{2\rho\sin\theta}{1-2\rho^2+\rho^2} = \pm\frac{2\rho\sqrt{1-\rho^2}}{1-2\rho^2+\rho^2} = \boxed{\pm\frac{2\rho}{\sqrt{1-\rho^2}} = \tilde{X}_a}$

$\begin{cases} + \text{ si } 0 \le \theta_1 \le \pi/2 \\ - \text{ si } -\frac{\pi}{2} \le \theta_2 \le 0 \end{cases}$



② ex: $R_L = 2Z_0$, $r_L = \frac{1}{3} \Rightarrow \rho = \frac{1}{3}$, $\psi = 0$

$\Rightarrow \theta_1 = 1.23$ ou $\theta_2 = -1.23$

$\Rightarrow z_1 = 0.0979\lambda$ ou $z_1' = -0.098$ ✓

$-\frac{\lambda}{2}$ ↳ $z_1 = -0.402\lambda$ ✓

$\tilde{X}_a = 0.707$
$z_1 = -0.402\lambda$
$\Rightarrow$ $-j\tilde{X}_a$ en z_1

$\tilde{X}_a' = -0.707$
$z_1' = -0.098$
$\Rightarrow$ $-j\tilde{X}_a'$ en z_1'

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C58 - Exercice

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{110-50}{110+50} = 0.375 \Rightarrow \rho = 0.375, \quad \psi = 0$$

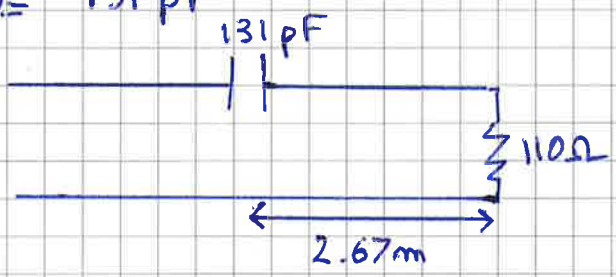
$$\lambda = \frac{2 \cdot 10^8}{30 \cdot 10^6} = 6.67\,m$$

$$\cos^{-1}(\rho) = \theta \Rightarrow \theta_1 = 1.186, \quad \theta_2 = -1.186$$

Première solution : $\frac{z_1}{\lambda} = 0.0944 - 0.5 = -0.4 \Rightarrow z_1 = -2.67\,m$

$$\tilde{X}_a = 0.81 \Rightarrow X_a = 40.45\,\Omega$$

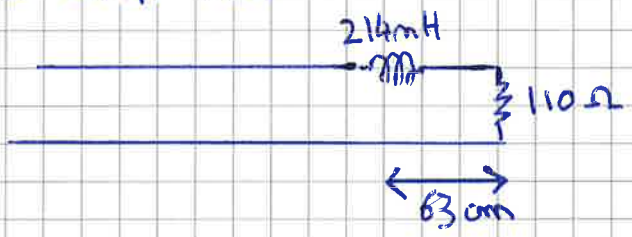
$$\Rightarrow C = (X_a \omega)^{-1} = 131\,pF$$



Deuxième solution $\frac{z_1'}{\lambda} = -0.0944 \Rightarrow z_1 = -0.63\,m$

$$\tilde{X}_a = -0.81 \Rightarrow X_a = -40.45\,\Omega$$

$$\Rightarrow L = \frac{X_a}{\omega} = 214\,mH$$



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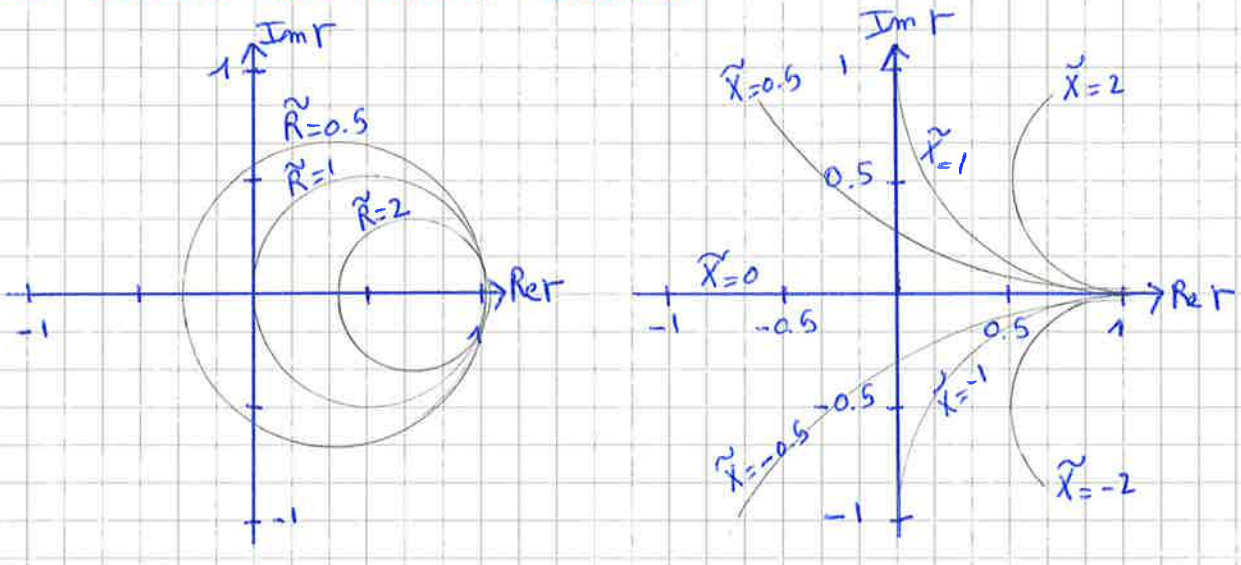
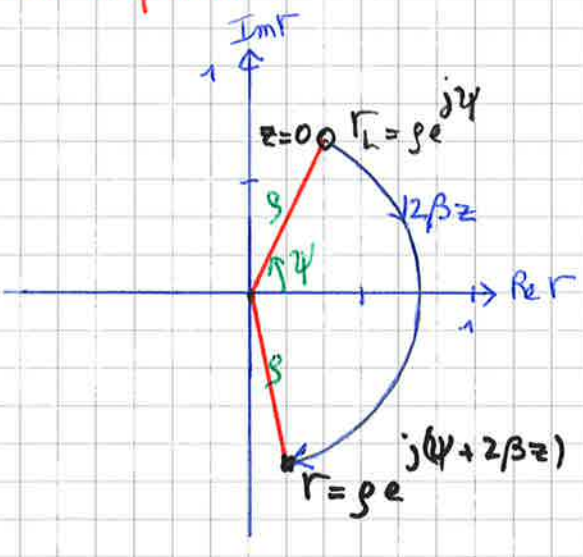
f) Abaque de Smith

Voir présentation powerpoint en cours et sur Moodle

C59 - Contours avec $\tilde{R}$ ou $\tilde{X}$ constants



C60 - Coefficient de réflexion Γ



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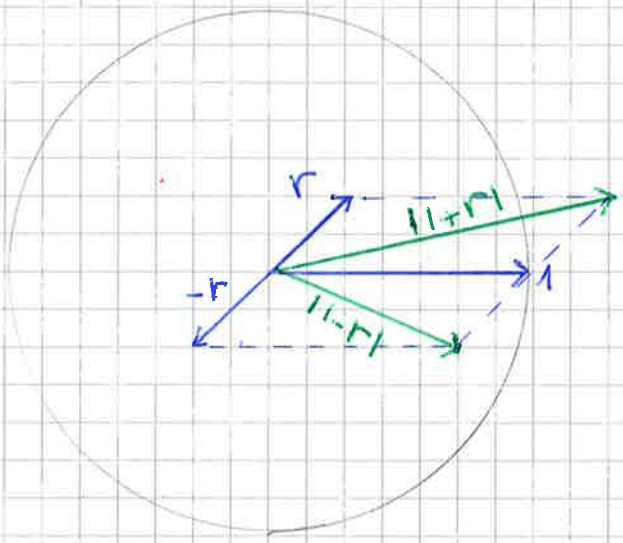
C61 - Tension et courant lus sur l'Abaque



$$\begin{cases} |I(z)| = \dfrac{|V^+|}{Z_0}\,|1-r| \\ |V(z)| = |V^+|\,|1+r| \end{cases}$$

C62 - Schéma pour l'exemple 5